Codified procedure for buffeting response of buildings and bridges

Kristoffer Hoffmann [a], Marie Louise Pedersen [a], Svend Ole Hansen [a,b,c]

[a]*Svend Ole Hansen ApS, Sct. Jørgens Allé 5C, Copenhagen, Denmark*
[b]*SOH Wind Engineering LLC, 141 Leroy Rd, Williston, Vermont, USA*
[c]*Leader of the Project Team revising the Eurocode EN 1991-1-4 on wind actions*

ABSTRACT: Codified specifications on wind actions, such as those prescribed in the Eurocode EN 1991-1-4, ISO, and ASCE norms, contain various procedures for calculating the buffeting response of typical buildings and bridge structures. Most often such procedures provide a simple, robust, and operational framework allowing for accurate predictions of the relevant wind loads and responses for many types of structures. The codified procedures for calculating the buffeting response are, however, not always applicable in the case of complex loading scenarios, such as the resonant response of structures with mode shapes of non-constant sign, and some of the prescribed mathematical expressions may contain coefficients and variables that are difficult to interpret in a physically consistent manner.

The present document outlines a new codified procedure for calculating the along-wind buffeting response of buildings and bridges. The new procedure is simple and operational, and extends the current Eurocode EN 1991-1-4 provisions to cover mode shapes with non-constant signs, allows for the systematic use of cross-sectional admittance functions, and ensures an asymptotically consistent modeling of the two-dimensional surface pressure characteristics. The perspective of the new calculation procedure is also discussed to reflect on the implications of updating the buffeting response calculation procedure in the next revision of the Eurocode.

KEYWORDS: Eurocode, Buffeting load, Buffeting response, Size-reduction function, Joint-acceptance function, Aerodynamic admittance function

1 INTRODUCTION

The current Eurocode EN 1991-1-4[1] provides two alternative procedures for calculating the dynamic along-wind buffeting response of structures. The two procedures provide predictions of the same order of magnitude, and both procedures assume that:

1. the wind load is determined from the undisturbed wind field;
2. the structure is linear-elastic with viscous damping;
3. the along-wind mode considered is uncoupled from other modes.

According to assumption 1 above, the pressure correlation is assumed to be identical to the correlation of the longitudinal turbulence in the undisturbed wind field. This assumption may lead to an overestimation of the load, due to lack of correlation between wind load on the structure front and on the rear face. On the other hand the load will be underestimated due to the fact that pressures on the structure are better correlated than the longitudinal turbulence in the undisturbed wind field.

The basis of the procedure outlined in the present document is a cross-sectional admittance function, which gives the conversion between the spectrum of the incoming air flow and the spectrum of the wind action on a given structural cross section. The cross-sectional admittance function is subsequently combined with the correlation of the wind flow for longitudinal turbulence component separated along the main axis of the structure. For a line-like structure the correlation between cross sectional forces will be described well by the correlation of the incoming flow with the same separation. The procedure also allows for a more direct application of the inertial forces in the determination of the structural response.

2 STRUCTURAL RESPONSE DUE TO BUFFETING

The fluctuating part of the structural response consists of two main contributions, originating from background turbulence, i.e. the broadband turbulence fluctuations in the wind, and resonance turbulence, i.e. the turbulence fluctuations in the wind in resonance with a natural frequency of the structure. In the present Eurocode format, their respective contributions are expressed by statistical variance terms in the response, ultimately prescribing the characteristic response.

It is necessary to include the integral effect of response reduction caused by the lack of full surface pressure correlation to avoid an unnecessary overestimation of the background turbulence response on large structures. On the other hand, to avoid an underestimation of the resonant response on dynamically sensitive structures, the resonant amplification of turbulent loads near a structural natural frequency should be included[2]. In the Eurocode format, the relative effect of these two wind effects are modelled by the background response factor B^2 and the resonant response factor R^2, respectively. The theory presented in the following subsections is based on the Davenport wind load model[2].

2.1 *Quasi-static peak wind force model*

Consider a structure of width b and height l, where the mean wind direction is directed towards a plate-like surface of coordinates $y \in [0, b]$ and $z \in [0, l]$; see Figure 1.

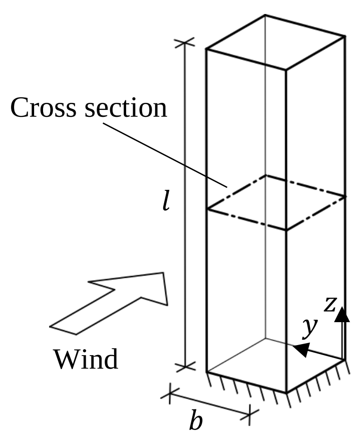

Figure 1. Example of model structure of width b and height l.

For each point on the surface, let the fluctuating wind load be defined by the sum

$$F(y, z, t) = F_q(y, z) + F_t(y, z, t),$$

where F_q is the mean wind point load and F_t is the fluctuating part of the point load due to wind turbulence. The mean and fluctuating part may be modelled by[2]

$$F_q(y,z) = \frac{1}{2}\rho\, v_\mathrm{m}(y,z)^2\, c_\mathrm{f}(y,z), \text{ and } F_\mathrm{t}(y,z,t) = \rho\, v_\mathrm{m}(y,z)\, u(y,z,y) c_\mathrm{f}(y,z),$$

where ρ is the air density, v_m is the characteristic mean wind velocity, u is the along-wind turbulence component, and c_f is a force coefficient for the load considered.

The mean response, which originates from the mean wind load acting on the structure, is then

$$\mu_\mathrm{R} = \int_0^l \int_0^b F_q(y,z) I_\mathrm{R}(y,z)\, dy\, dz,$$

where $I_R(y,z)$ is the response influence function equal to the response obtained for a unit load acting at the point (y,z). As an example, $I_\mathrm{R}(y,z) = z$ in order to evaluate the bending moment at $z = 0$.

The background turbulent response is calculated by treating the fluctuating wind load caused by turbulence F_t in the quasi-static fashion

$$R_\mathrm{b}(t) = \int_0^l \int_0^b F_\mathrm{t}(y,z,t) I_\mathrm{R}(y,z)\, dy\, dz.$$

The resonant turbulent response may be calculated using modal analysis. The resonant response is usually dominated by a fundamental mode and the corresponding generalized fluctuating load

$$Q(t) = \int_0^l \int_0^b F_\mathrm{t}(y,z,t) \xi(y,z)\, dy\, dz,$$

where $\xi(y,z)$ is the non-dimensional deflection mode shape.

2.2 *Evaluation of responses as stochastic processes*

The fluctuating part of the load due to wind turbulence may be treated as a stochastic process. This gives the following expression for the variance of the background response[3]

$$\sigma_\mathrm{b}^2 = \int_0^l \int_0^l \int_0^b \int_0^b g_\mathrm{b}(y_1,z_1) g_\mathrm{b}(y_2,z_2) \rho_u(r_y,r_z)\, dy_1 dy_2\, dz_1 dz_2,$$

where $g_\mathrm{b}(y,z) = \rho v_\mathrm{m}(y,z) c_\mathrm{f}(y,z) I_\mathrm{R}(y,z) \sigma_u(y,z)$. The function $\rho_u(r_y,r_z)$ is the correlation coefficient for the along-wind turbulence component separated by $r_y = |y_2 - y_1|$ and $r_z = |z_2 - z_1|$.

The dynamic part of the deflection may as an approximation be expressed as $a(t)\xi(x,y)$, where $a(t)$ is a stochastic amplitude function. The spectral density of $a(t)$ is then given by

$$S_a(n) = |H_i(n)|^2 S_Q(n),$$

where $H_i(n)$ is the frequency response function associated with mode i and the natural frequency n_i.

Let the power spectrum of the along-wind turbulence be denoted by S_u. The generalized load spectrum is then[3]

$$S_Q(n) = \int_0^l \int_0^l \int_0^b \int_0^b g_\mathrm{r}(y_1,z_1,n) g_\mathrm{r}(y_2,z_2,n) \psi_u(r_y,r_z,n,v_\mathrm{m})\, dy_1 dy_2\, dz_1 dz_2,$$

where $g_\mathrm{r}(y,z) = \rho v_\mathrm{m}(y,z) c_\mathrm{f}(y,z) \xi(y,z) \sqrt{S_u(y,z,n)}$. The function $\psi_u(r_y,r_z,n,v_\mathrm{m})$ is the normalized co-spectrum for the along-wind turbulence components separated by r_y and r_z. The variance of the stochastic amplitude function $a(t)$ is then found by an integration of the spectral density, i.e.

$$\sigma_a^2 = \int_0^\infty |H_i(n)|^2 S_Q(n)\, dn.$$

The damping consists of both aerodynamic and structural damping, and is often relatively low, meaning $\zeta_i \ll 1$, and $S_Q(n)$ usually has most of its values at frequencies below n_i. In this case the so-called white noise approximation may be applied[3]

$$\sigma_a^2 \approx S_Q(n_i) \int_0^\infty |H_i(n)|^2\, dn \approx \frac{\pi^2}{2\delta_i} \frac{1}{m_\mathrm{G}^2} \frac{n_i}{(2\pi n_i)^4} S_Q(n_i),$$

where the last integral is found by contour integration and m_G denotes the modal mass. The damping is here expressed by a logarithmic decrement, i.e. $\delta_i = 2\pi\zeta_i$, valid for small damping ratios. The fluctuating loads due to resonant buffeting may then be expressed by inertia forces proportional to the acceleration, and the variance of the resonant response becomes

$$\sigma_\mathrm{R}^2 = \left(\int_0^l \int_0^b \mu(y,z) \xi(y,z)\, I_\mathrm{R}(y,z)\, dy\, dz \right)^2 (2\pi n_i)^4\, \sigma_a^2 = \frac{\pi^2}{2\delta_i}\, K_\mathrm{m}^2\, n_i\, S_Q(n_i),$$

where the load-response parameter K_m is defined as

$$K_\mathrm{m} = \frac{\int_0^l \int_0^b \mu(y,z) \xi(y,z) I_\mathrm{R}(y,z)\, dy\, dz}{m_\mathrm{G}} = \frac{\int_0^l \int_0^b \mu(y,z) \xi(y,z) I_\mathrm{R}(y,z)\, dy\, dz}{\int_0^l \int_0^b \mu(y,z) \xi^2(y,z)\, dy\, dz}.$$

Here $\mu(y,z)$ denotes the mass per unit area of the structure considered. The load-response parameter is proportional to the ratio between the generalized load and the response calculated using inertia forces. The unit of K_m is equal the unit of I_R.

2.3 *Background and resonant response factor*

The standard deviation of the structural response is assumed to be the sum of the uncorrelated contributions from the background and resonant turbulence response. Allowing different peak factors for the two contributions gives the following relation for the characteristic response

$$R_\mathrm{max} = \mu_\mathrm{R} \pm \sqrt{(k_\mathrm{B} \cdot \sigma_\mathrm{B})^2 + \left(k_\mathrm{p} \cdot \sigma_\mathrm{R}\right)^2}.$$

The characteristic response may be defined relative to the mean response using the along-wind turbulence intensity, $I_u = \sigma_u / U$, by the expression

$$R_\mathrm{max} = \mu_\mathrm{R} \left(1 \pm 2 \cdot I_u \cdot \sqrt{(k_\mathrm{B} \cdot B)^2 + \left(k_\mathrm{p} \cdot R\right)^2} \right), \tag{1}$$

where

$$B^2 = \frac{\sigma_\mathrm{B}^2}{4 I_u^2 \cdot \mu_\mathrm{R}^2}, \quad R^2 = \frac{\sigma_\mathrm{R}^2}{4 I_u^2 \cdot \mu_\mathrm{R}^2}.$$

The parameters B^2 and R^2 are denoted the background response factor and the resonant response factor, respectively. The following section will explain how B^2 and R^2 may under certain assumptions be evaluated in a relatively simple approximate format, even though they are represented mathematically by rather complex quadruple integrals; see Eq. (6) and (7).

Note that it is not always possible to express the background and resonant turbulence response using the mean response, for instance when the mean response is zero or for response influence functions and structural mode shapes with a non-constant sign. This will be discussed further in Section 5.

3 PRODUCT FORMAT FOR DOUBLE AND QUADRUPLE INTEGRALS

For simplicity, it is in the following assumed that the force coefficient c_{f}, the characteristic mean wind velocity v_{m}, the wind turbulence power spectrum S_u, and the along-wind variance σ_u^2 are constant, or at least evaluated at a single representative point on the structure. Furthermore, the correlation and co-spectral properties are modelled by two-dimensional exponential expressions[3,4]

$$\rho_u(r_y, r_z) = \exp\left(-\sqrt{\left(\frac{r_y}{L_u^y}\right)^2 + \left(\frac{r_z}{L_u^z}\right)^2}\right), \quad \psi_u(r_y, r_z, n_i, v_{\mathrm{m}}) = \exp\left(-\frac{n_i}{v_{\mathrm{m}}}\sqrt{\left(c_y r_y\right)^2 + (c_z r_z)^2}\right),$$

where L_u^y, L_u^z are the integral length scales, and c_y, c_z are decay constants for the normalized co-spectrum of the along-wind turbulence components associated with separations along y and z. Finally, it is also assumed that the two-dimensional response influence functions and mode shapes may be expressed as product of two one-dimensional functions, i.e. $I_{\mathrm{R}}(y,z) = I_{\mathrm{R},y}(y) I_{\mathrm{R},z}(z)$ and $\xi(y,z) = \xi_y(y)\xi_z(z)$.

Let the non-dimensional power spectral density function be defined by $S_{\mathrm{N},u}(n) = \frac{n S_u(n)}{\sigma_u^2}$. Then the expressions for B^2 and R^2 takes the form

$$B^2 = \frac{\int_0^l \int_0^l \int_0^b \int_0^b I_{\mathrm{R},y}(y_1) I_{\mathrm{R},z}(z_1) I_{\mathrm{R},y}(y_2) I_{\mathrm{R},z}(z_2) \rho_u(r_y, r_z)\, dy_1\, dy_2\, dz_1\, dz_2}{\left(\int_0^l \int_0^b I_{\mathrm{R},y}(y) I_{\mathrm{R},z}(z)\, dy\, dz\right)^2}, \tag{2}$$

$$R^2 = \frac{\pi^2}{2\delta_i} S_{\mathrm{N},u}(n_i) K_{\mathrm{m}}^2 \frac{\int_0^l \int_0^l \int_0^b \int_0^b \xi_y(y_1)\xi_z(z_1)\xi_y(y_2)\xi_z(z_2)\psi_u(r_y, r_z, n_i, v_{\mathrm{m}})\, dy_1\, dy_2\, dz_1\, dz_2}{\left(\int_0^l \int_0^b I_{\mathrm{R},y}(y) I_{\mathrm{R},z}(z)\, dy\, dz\right)^2}. \tag{3}$$

Note that both expressions consist of quadruple integrals of response influence function or mode shapes multiplied by an exponential term.

3.1 *Asymptotic representation of quadruple integrals as double integrals*

The evaluation of the quadruple integrals given in Eq. (2) and (3) may be greatly simplified by utilizing an approximate representation with correct asymptotic behavior for situations corresponding to a large structural dimension compared to the integral length scale, for the background turbulence case, or corresponding to a large dimension compared to the average size of the turbulent vortices, for the resonant turbulent case.

Denote the ratios between the structural dimensions and integral length scales by $\phi_y = b/L_u^y$ and $\phi_z = l/L_u^z$. Then the following asymptotic limit is obtained[3]

$$\lim_{\phi_y \to \infty, \phi_z \to \infty} B^2 = \frac{\pi}{2} \cdot \left(\lim_{\phi_y \to \infty} B_y^2\right) \cdot \left(\lim_{\phi_z \to \infty} B_z^2\right), \tag{4}$$

where

$$B_y^2(\phi_y) = \frac{\int_0^b \int_0^b I_{\mathrm{R},y}(y_1) I_{\mathrm{R},y}(y_2) \exp\left(-\phi_y \frac{r_y}{b}\right) dy_1 dy_2}{\left(\int_0^b I_{\mathrm{R},y}(y)\, dy\right)^2},$$

and similar for $B_z^2(\phi_z)$. The factor $\pi/2$ is a purely mathematical term representing the ratio between the product of plate-like and line-line integral functions in the asymptotic limit.

The mathematical simplification presented above is much similar for the resonant response factor, except that $\phi_y = bc_y n_i / v_\mathrm{m}$ and $\phi_z = lc_z n_i / v_\mathrm{m}$, and the response influence function in the numerator is replaced by the mode shape, i.e.

$$\lim_{\phi_y\to\infty,\phi_z\to\infty} R^2 = \frac{\pi^2}{2\delta_i} S_{\mathrm{N},u}(n_i) \cdot \frac{\pi}{2} \cdot K_\mathrm{m}^2 \cdot \left(\lim_{\phi_y\to\infty} R_y^2\right) \cdot \left(\lim_{\phi_z\to\infty} R_z^2\right), \tag{5}$$

where

$$R_y^2(\phi_y) = \frac{\int_0^b \int_0^b \xi_y(y_1)\xi_y(y_2) \exp\left(-\phi_y \frac{r_y}{b}\right) dy_1 dy_2}{\left(\int_0^b I_{\mathrm{R},y}(y)\, dy\right)^2}.$$

and similar for $R_z^2(\phi_z)$.

3.2 *Analytic expression for uniform response influence function or mode shape*

It is the goal to make a simple analytic framework for evaluating integrals of the form similar to $B_y^2(\phi_y)$ and $R_y^2(\phi_y)$ for constant-sign response influence functions and mode shapes. The fundamental reference case is when the response influence function is uniform, i.e. $I_{\mathrm{R},y}(y) = 1$, for which the double integral has the analytic solution[3]

$$B_\mathrm{U}^2(\phi_y) = \frac{2}{\phi_y^2}\left(\phi_y - 1 + \exp(-\phi_y)\right).$$

The uniform reference case satisfies the asymptotic limit

$$\lim_{\phi_y\to\infty} B_\mathrm{U}^2(\phi_y) = \frac{2}{\phi_y}.$$

The asymptotic limit of $B_y^2(\phi_y)$ for a general response influence function is[3]

$$\lim_{\phi_y\to\infty} B_y^2(\phi_y) = \frac{2}{\phi_y} \frac{b \int_0^b I_{\mathrm{R},y}^2(y)\, dy}{\left(\int_0^b I_{\mathrm{R},y}(y)\, dy\right)^2}.$$

A scaling of the argument in the analytic expression associated with the uniform reference case may therefore be used to obtain an asymptotically correct analytic expression for B_y of the form

$$B_y^2(\phi_y) \approx B_\mathrm{U}^2(\alpha_{B,y} \cdot \phi_y), \text{ where } \alpha_{B,y} = \frac{\left(\int_0^b I_{\mathrm{R},y}(y) dy\right)^2}{b \int_0^b I_{\mathrm{R},y}^2(y) dy}.$$

For the background response, the correlation scaling factor $\alpha_{B,y}$ depends only on the response influence function, and it can be shown that $\lim_{\phi_y\to 0} B_y^2(\phi_y) = 1$ independent on the response influence function[2]. This means that the correct asymptotic behaviour for small values of ϕ_y is also obtained using the approximation. The same approximation may be applied to $B_z^2(\phi_z)$.

A similar approximation may be applied to the resonant response factor. Again, the aim is to express the double integral formula by the simple analytic expression for a uniform response influence function, $I_{\mathrm{R},y}(y) = 1$, and uniform mode shape, $\xi_y(y) = 1$, i.e.

$$R_\mathrm{U}^2(\phi_y) = \frac{2}{\phi_y^2}\left(\phi_y - 1 + \exp(-\phi_y)\right),$$

for which the asymptotic limit is given by

$$\lim_{\phi_y\to\infty} R_\mathrm{U}^2(\phi_y) = \frac{2}{\phi_y}.$$

The asymptotic limit of $R_y^2(\phi_y)$ for a general type of mode shape is

$$\lim_{\phi_y\to\infty} R_y^2(\phi_y) = \frac{2}{\phi_y}\frac{b\int_0^b \xi_y^2(y)\,dy}{\left(\int_0^b I_{\mathrm{R},y}(y)\,dy\right)^2}.$$

An asymptotically correct analytic expression for the integral is therefore given by

$$R_y^2(\phi_y) \approx R_{\mathrm{U}}^2(\alpha_{R,y}\cdot\phi_y), \text{ where } \alpha_{R,y} = \frac{\left(\int_0^b I_{\mathrm{R},y}(y)dy\right)^2}{b\int_0^b \xi_y^2(y)dy}.$$

The correlation scaling factor $\alpha_{R,y}$ depends on the mode shape and the response influence function. Due to the asymptotic behaviour of R_{U}^2, it is also possible to include the effect of the load-response parameter K_{m} using an additional scaling of the argument, i.e.

$$K_{\mathrm{m}}^2\cdot R_y^2(\phi_y) \approx R_{\mathrm{U}}^2(\alpha_{R,y}\cdot\phi_y), \text{ where } \alpha_{R,y} = \frac{\int_0^b \xi_y^2(y)dy\cdot\left(\int_0^b I_{\mathrm{R},y}(y)dy\right)^2}{b\left(\int_0^b \xi_y(y)I_{\mathrm{R},y}(y)dy\right)^2}.$$

The same approximation may be applied to $R_z^2(\phi_z)$, but that the scaling using K_{m} should only be applied to either $R_y^2(\phi_y)$ or $R_z^2(\phi_y)$.

The presented approximation for the resonant response factor does not necessarily ensure a correct asymptotic behaviour for $\phi_y\to 0$. In the uniform reference case, the limit for $\phi_y\to 0$ is unity, while the correct asymptotic value is

$$\lim_{\phi_y\to 0}\left(K_{\mathrm{m}}^2\cdot R_y^2(\phi_y)\right) = K_{\mathrm{m}}^2\cdot\frac{\left(\int_0^b \xi_y(y)\,dy\right)^2}{\left(\int_0^b I_{\mathrm{R},y}(y)\,dy\right)^2}.$$

This shows that the correct asymptotics are obtained for $\phi_y\to 0$ if the mode shape and response influence functions are of similar form, e.g. both uniform or both linear.

The additional scaling of the argument provides a great simplification of the calculation format. The resonant response factor is for typical structures often evaluated at relatively large values of ϕ_y, where the approximation turns out to be fairly precise, see also Section 4.2.

The idea of using a uniform reference influence response function or mode shape to develop simple analytic expressions for evaluating general background and resonant response factors is also a part of the theory behind Procedure 1 in the current Eurocode, which was developed by Prof. G. Solari, University of Genoa.

3.3 *Product format for response influence function and mode shapes of constant sign*

Consider a tall structure where $l > b$ and assume that the response influence function and mode shape does not depend on y, i.e. $I_{\mathrm{R},y}(y) = \xi_y(y) = 1$. Let the considered response be the bending moment at $z = 0$, corresponding to the response influence function $I_{\mathrm{R},z}(z) = z$. This is a model scenario covering the along-wind response of a typical tall building. In the following text, the subscript "L" is used to denote quantities associated with the main structural dimension.

Define the one-dimensional admittance function for constant sign response influence function and mode shapes of constant sign via the uniform reference case, i.e.

$$\chi_{\mathrm{U}}^2(\phi) = \frac{2}{\phi^2}(\phi - 1 + \exp(-\phi)).$$

Based on the derivation presented previously in this section, the background response factor may then be approximated by

$$B^2 \approx \chi_{\mathrm{U}}^2\left(\frac{2}{\pi}\cdot\phi_y\right)\cdot\chi_{\mathrm{U}}^2(\alpha_{B,\mathrm{L}}\cdot\phi_z), \text{ where } \alpha_{B,\mathrm{L}} = \frac{\left(\int_0^l I_{\mathrm{R},z}(z)dz\right)^2}{l\int_0^l I_{\mathrm{R},z}^2(y)dz} = \frac{3}{4}. \tag{6}$$

This factor is to be utilized in Eq. (1). Note that the admittance combination factor of $2/\pi$ is included by a scaling of the argument of the admittance function associated with the smallest structural dimension. This ensures that the correct asymptotic behavior of the product format is also obtained for in the asymptotic limit of line-line structures, since $\lim_{\phi\to 0}\chi_{\mathrm{U}}^2\left(\frac{2}{\pi}\cdot\phi\right) = 1$.

Similar to the approximation of the background response factor, the resonant response factor may be approximated by

$$R^2 \approx \frac{\pi^2}{2\delta_i}S_{\mathrm{N},u}(n_i)\cdot\chi_{\mathrm{U}}^2\left(\frac{2}{\pi}\cdot\phi_y\right)\cdot\chi_{\mathrm{U}}^2(\alpha_{R,\mathrm{L}}\cdot\phi_z), \text{ where } \alpha_{R,\mathrm{L}} = \frac{\int_0^l \xi_z^2(z)dz\cdot\left(\int_0^l I_{\mathrm{R},z}(z)dz\right)^2}{l\left(\int_0^l \xi_z(z)I_{\mathrm{R},z}(z)dz\right)^2}. \tag{7}$$

This factor is to be utilized in Eq. (1).

Mode shapes are often approximated by an exponential expression of the form $\xi_z(z) = (z/l)^\zeta$, where $\zeta \geq 0$. The uniform ($\zeta = 0$), linear ($\zeta = 1$), and parabolic ($\zeta = 2$) mode shapes are examples. The correlation scaling factor $\alpha_{R,\mathrm{L}}$ is for such an expression given by

$$\alpha_{R,\mathrm{L}} = \frac{\int_0^l\left(\frac{z}{l}\right)^{2\zeta}dz\cdot\left(\int_0^l I_{\mathrm{R},z}(z)\,dz\right)^2}{l\left(\int_0^l\left(\frac{z}{l}\right)^{\zeta} I_{\mathrm{R},z}(z)\,dz\right)^2} = \frac{1}{4}\cdot\frac{(\zeta+2)^2}{2\cdot\zeta+1}.$$

The procedure outlined above illustrates the general applicability of the product format.

3.4 *Characteristic along-wind acceleration*

The characteristic peak acceleration of a structure is relevant for occupant comfort. Adopting the definitions presented in the previous subsections, the variance of the peak structural acceleration is given by

$$\sigma_{\mathrm{acc}}^2 = (2\pi n_i)^4\,\sigma_a^2 = R^2\cdot\frac{4I_u^2\cdot\mu_{\mathrm{R}}^2}{\left(\int_0^l\int_0^b \mu(y,z)\xi(y,z)\,I_R(y,z)\,dy\,dz\right)^2}.$$

The standard deviation scales with the mode shape and may be expressed by

$$\sigma_{\mathrm{acc}}(z) = 2\cdot c_{\mathrm{f}}\cdot I_u\cdot q_{\mathrm{m}}\cdot b\cdot K_{R,\mathrm{L}}\cdot R\cdot\frac{\xi_z(z)}{m_{\mathrm{e}}\cdot\xi_{\max}},$$

where $\xi_{\max}$ is the mode shape value at the point with maximum amplitude, m_{e} denotes a reference mass per unit length, and the mean wind velocity pressure q_{m} and the load distribution factor $K_{R,\mathrm{L}}$ are defined as

$$q_{\mathrm{m}} = \frac{1}{2}\rho v_{\mathrm{m}}^2, \quad K_{R,\mathrm{L}} = \frac{\int_0^l I_{\mathrm{R}}(z)\,dz}{\int_0^l \xi_z(z)I_{\mathrm{R}}(z)\,dz}.$$

For mode shapes given by an exponential expression of the form $\xi_z(z) = \left(\frac{z}{l}\right)^\zeta$, where $\zeta \geq 0$, the load distribution factor $K_{R,\mathrm{L}}$ is given by

$$K_{R,\mathrm{L}} = \frac{\int_0^l I_{\mathrm{R}}(z)\,dz}{\int_0^l\left(\frac{z}{l}\right)^\zeta I_{\mathrm{R}}(z)\,dz} = \frac{1}{2}\cdot(\zeta+2).$$

For mode shapes of constant sign, the load distribution factor K_R is the scaling of a load distribution proportional to the mode shape resulting in a response identical to that of a uniform load distribution, see Figure 2.

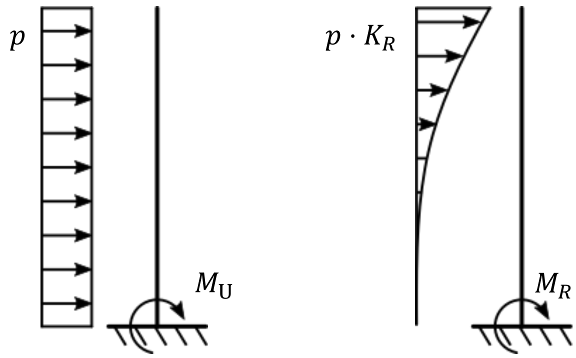

Figure 2. For mode shapes of constant sign, the load distribution factor K_R is the scaling of a load distribution proportional to the mode shape resulting in a response identical to that of a uniform load distribution, i.e. $M_{\mathrm{U}} = M_R$.

4 ASYMPTOTIC BEHVAVIOUR OF PRODUCT FORMAT

The following section illustrates the asymptotic behavior of the proposed product format. The model scenario is again the along-wind response of a tall building, i.e. $l > b$ and a response influence function and mode shape that do not depend on y, i.e. $I_{\mathrm{R},y}(y) = \xi_y(y) = 1$, Also, let the response be the bending moment at $z = 0$, corresponding to $I_{\mathrm{R},z}(z) = z$.

4.1 *Background response factor*

The background response factor B^2 determined using the quadruple integral (Eq. (2)), using the product of two double integrals (Eq. (4)), and using the proposed product format (Eq. (6)) are all presented in Figure 3, for $\phi_y = 0$ and $\phi_y = 0.5 \cdot \phi_z$. The two scenarios correspond to a line-like structure and a structure where $b \approx 0.5 \cdot l$, respectively.

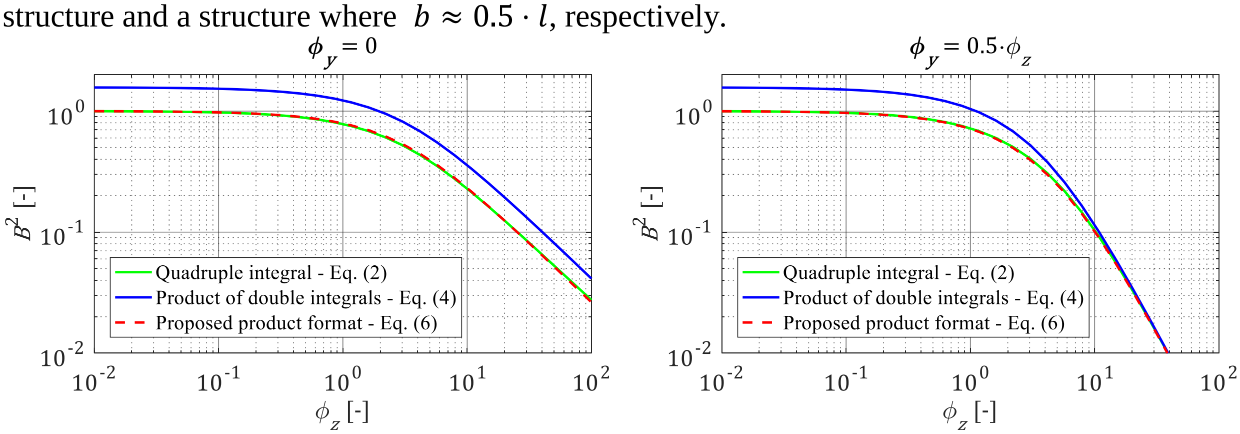

Figure 3. The background response factor B^2 determined by numerical integration of the quadruple integral (Eq. (2)), using the product of two double integrals (Eq. (4)), and using the proposed product format (Eq. (6)). The plots consider $\phi_y = 0$ (left figure) and $\phi_y = 0.5 \cdot \phi_z$ (right figure).

Figure 3 illustrates that the proposed product format is a very good approximation of the quadruple integral for all values of ϕ_z. The admittance combination factor of $\pi/2$ is clearly necessary to include to ensure a correct asymptotic behavior when both ϕ_y and ϕ_z approach infinity. When both ϕ_y and ϕ_z approach zero, the product of double integrals (Eq. (4)) approaches $\pi/2$.

4.2 *Resonant response factor*

Let the mode shape along the z dimension be parabolic, i.e. $\xi_z(z) = (z/l)^2$. The resonant response factor R^2 determined using the quadruple integral (Eq. (3)), using the product of two double integrals (Eq. (5)), and using the proposed product format (Eq. (7)) are all presented in Figure 4, using the expression $R^2/(\frac{\pi^2}{2\delta_i} S_{\mathrm{N},u}(n_i))$, for $\phi_y = 0$ and $\phi_y = 0.5 \cdot \phi_z$. The plots illustrate that the proposed product format is a very good approximation of the quadruple integral for $\phi_z \geq 10$. The admittance combination factor of $\pi/2$ is clearly necessary to include to ensure a correct asymptotic behavior when both ϕ_y and ϕ_z approach infinity. For $\phi_z < 10$ the proposed format is conservative.

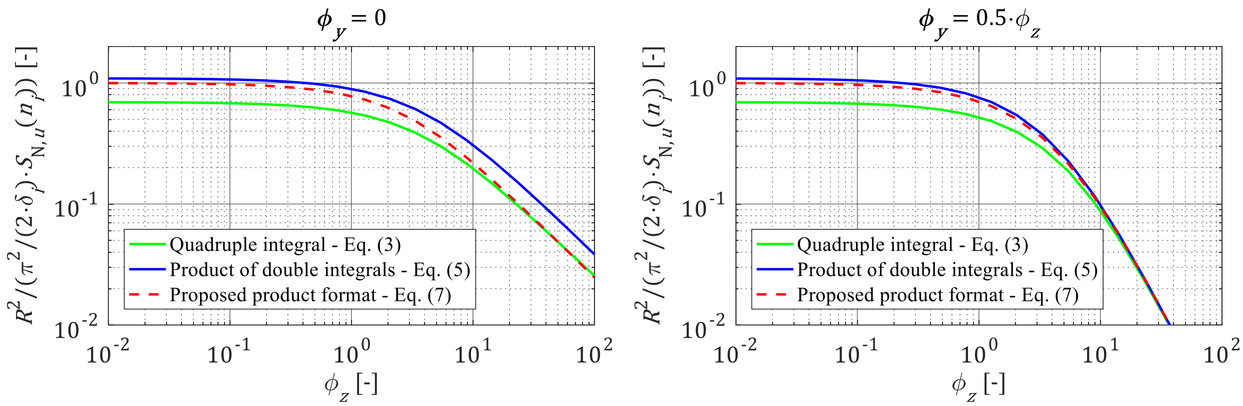

Figure 4. The expression $R^2/(\frac{\pi^2}{2\delta_i} S_{\mathrm{N},u}(n_i))$ evaluated for a parabolic mode shape determined by numerical integration of the quadruple integral (Eq. (3)), using the product of two double integrals (Eq. (5)), and using the proposed product format (Eq. (7)). The plots consider $\phi_y = 0$ (left figure) and $\phi_y = 0.5 \cdot \phi_z$ (right figure).

For a typical type of structure covered by the present Eurocode, the natural frequency in Hertz of may be approximated by $n_i \approx 50/l$, and adopting typical values of the decay constant, $c_z = 10$, and the characteristic mean wind velocity, $v_{\mathrm{m}} = 25$ m/s, implies that $\phi_z \approx 20$. This underlines that typical structures correspond to situations where the proposed product format is a very good approximation.

As seen in Figure 4, the three different formula has different asymptotic limits for $\phi_z \to 0$, see also Section 3.2. The quadruple integral approaches $(5/6)^2$, the product of double integrals approaches $\pi/2 \cdot (5/6)^2$, while the proposed product format approaches 1.

Note that the comparison of the resonant response factor for a linear mode shape, i.e. $\xi_z(z) = z/l$, is equivalent to the results presented for the background response factor in the previous subsection, since the response influence function is assumed linear.

5 REPONSE INFLUENCE FUNCTIONS AND MODE SHAPES WITH CHANGING SIGNS

For non-constant sign response influence functions or mode shapes, it is not possible to express the fluctuating part of the structural response in terms of the mean response. Instead, the fluctuating part of the structural response may be expressed using a non-uniform reference load distribution for the background response and by equivalent static loads for the resonant response. The aerodynamic admittance functions may then be expressed using their analytic expressions, which exist for several fundamental structure types, such as cantilever and simply supported structures. The mode shapes associated with these fundamental structures are illustrated in Figure 5.

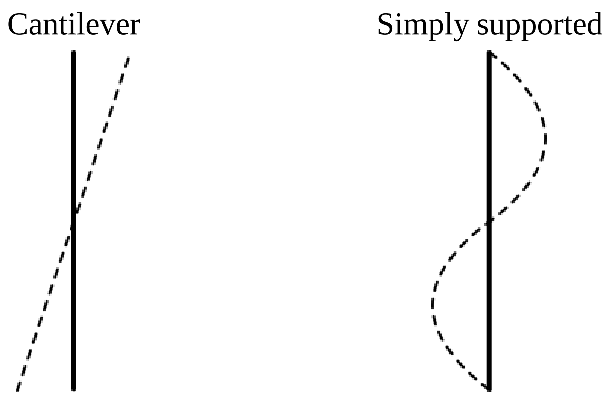

Figure 5. Mode shapes associated with a cantilever and a simply supported structure with three supports.

For simplicity, only the resonant response factor is considered in the following, but the principles are equivalent for the background response factor.

Let the model structure have its main direction along z, and a uniform response influence function and mode shape along y. The characteristic response due to resonant turbulence alone is then

$$R_{\mathrm{max},R} = k_{\mathrm{p}} \cdot 2 \cdot c_{\mathrm{f}} \cdot I_u \cdot q_{\mathrm{m}} \cdot b \cdot l \cdot K_{\mathrm{m}} \cdot R,$$

where the resonant response factor is

$$R^2 = \frac{\pi^2}{2\delta_i} S_{\mathrm{N},u}(n_i) \cdot \chi_{\mathrm{U}}^2\left(\frac{2}{\pi} \cdot \phi_y\right) \cdot \chi_{\mathrm{L}}^2(\phi_z).$$

The analytic expressions of the one-dimensional admittance function for the cantilever structure is

$$\chi_{\mathrm{L}}^2(\phi) = \frac{2}{3 \cdot \phi} - \frac{2}{\phi^2} + \frac{8}{\phi^4} - e^{-\phi}\left(\frac{2}{\phi^2} + \frac{8}{\phi^3} + \frac{8}{\phi^4}\right),$$

and for a simply supported structure with three supports it is

$$\chi_{\mathrm{L}}^2(\phi) = \frac{e^{-\phi}}{(4 \cdot \pi^2 + \phi^2)^2}\left(e^{\phi} \cdot (4 \cdot \pi^2 \cdot (\phi + 2) + \phi^3) - 8 \cdot \pi^2\right).$$

The fluctuating part of the structural response due to resonant turbulence may be expressed using a non-uniform reference load distribution defined relative to the mode shape, i.e.

$$F_{\mathrm{w}}(z) = F_{\mathrm{w,f}}\, \xi_z(z)/\xi_{\mathrm{max}},$$

where the maximum load per unit length is $F_{\mathrm{w,f}}$. The response associated with this load is

$$R_{\mathrm{max},R} = F_{\mathrm{w,f}} \int_0^l \xi_z(z) I_{\mathrm{R}}(z)\, dz.$$

The maximum amplitude of the fluctuating wind load per unit length may then be evaluated as

$$F_{\mathrm{w,f}} = 2 \cdot k_{\mathrm{p}} \cdot c_{\mathrm{f}} \cdot I_u \cdot q_{\mathrm{m}} \cdot b \cdot K_{R,\mathrm{L}} \cdot R,$$

where

$$K_{R,\mathrm{L}} = \frac{K_{\mathrm{m}}}{\frac{1}{l}\int_0^l \xi_z(z) I_{\mathrm{R}}(z)\, dz} = \frac{1}{\frac{1}{l}\int_0^l \xi_z^2(z)\, dz}.$$

Note that this expression for the load distribution factor $K_{R,\mathrm{L}}$ is not identical to the expression defined in Section 3.4 for a constant sign mode shape. For the cantilever model structure the load distribution factor becomes $K_{R,\mathrm{L}} = 3$ and for the simply supported model structure $K_{R,\mathrm{L}} = 2$.

Since the fluctuating part of the structural response due to resonant turbulence is expressed relative to the mode shape, the fluctuating wind load determined above is in principle an equivalent static load. This also implies that the maximum amplitude of the fluctuating wind load per unit length $F_{\mathrm{w,f}}$ does not depend on the response influence function.

6 PERSPECTIVE

One of the advantages of the new procedure is that the actual correlation of pressures and forces is taken into account via the cross-sectional admittance function. The current assumption of equivalence between wind pressure correlation and velocity correlation does not provide consistent results, and this non-consistency could be removed by the new approach proposed. The approach also facilitates the use of structure-specific wind load characteristics, such as aerodynamic admittance functions, determined directly from wind tunnel experiments.

The new procedure will relatively easily accommodate codified extensions in form of mode shapes with changing sign, across-wind buffeting response, and torsional buffeting response. The procedure described above has been applied successfully in buffeting response analyses for long-span cable-supported bridges. It is believed that the same approach may turn out to give a consistent and operational description of buffeting wind actions on buildings.

7 REFERENCES

1 EN1991-1-4, Eurocode 1: Actions on structures – General actions – Part 1-4: Wind actions, CEN, 2007.

2 A. G. Davenport, The response of slender, line-like structures to a gusty wind. Proceedings of the Institution of Civil Engineers, 23(3), 389–408, 1962.

3 C. Dyrbye and S.O. Hansen, Wind Loads on Structures, John Wiley & Sons Ltd, 1997.

4 S.O. Hansen and S. Krenk, Dynamic along-wind response of simple structures, Journal of Wind Engineering and Industrial Aerodynamics, 82, 1999.